GEOMETRIC CRITERIA FOR EVALUATING SKINNING DEFORMATION QUALITY OF STYLIZED 3D CHARACTERS: LOCAL VOLUME PRESERVATION, CURVATURE CONTINUITY AND A FOLD-FREE BLENDING CONDITION

Xabibullayeva (O'tkirbekova) Madinabonu Ravshanbek qizi
Tashkent University of Information Technologies
named after Muhammad al-Khwarizm
Tashkent, Uzbekistan.
E-mail: otkirbekovamadinabonu@gmail.com
Scientific supervisor: ***Kayumova Gulshan Asrorovna***
PhD (Ped.), Associate Professor

Abstract. Automatic rigging of stylized characters is usually judged visually, which makes results non-reproducible. This paper proposes a set of geometric criteria for skinning quality that is sensitive to joint defects: the local cross-sectional area deviation, a dihedral-angle measure of curvature continuity (creases and folds), and the reference-based Hausdorff and RMS errors. For a two-bone joint we derive closed-form expressions for the cross-section loss of linear blend skinning (LBS): $\cos(\theta/2)$ for bending and $\cos^2(\theta/2)$ for twisting. We also derive a fold-free condition for dual quaternion skinning (DQS): the half-width b of the blending zone must satisfy $b/r \geq 3\cdot\tan(\theta/4)$, where r is the limb radius and θ the joint angle. A computational experiment on 12,802-vertex tube meshes with radius-to-length ratios 0.10–0.45 confirms all three results to within the 0.05 sampling step. It also shows that global volume, the usual metric, underestimates joint collapse by a factor of 5–22 (5–8 for stylized limbs) and does not detect DQS folding at all. The condition gives an adaptive rule: stylized, thick limbs need proportionally wider blending zones.

Annotatsiya. Stilizatsiyalangan personajlarni avtomatik riglash natijasi odatda ko'z bilan baholanadi, bu esa natijani takrorlab bo'lmaydigan qiladi. Maqolada bo'g'im nuqsonlariga sezgir geometrik mezonlar tizimi taklif etiladi: kesim yuzasining mahalliy

og’ishi, ikki yoqli burchak bo’yicha egrilik uzluksizligi (burmalar va qatlanishlar), etalonga nisbatan Hausdorff va o’rtacha kvadratik xatoliklar. Ikki suyakli bo’g’im uchun chiziqli aralash skinning (LBS) kesim yo’qotishining yopiq ifodalari keltirib chiqarilgan: bukilishda $\cos(\theta/2)$, buralishda $\cos^2(\theta/2)$. Dual kvaternionli skinning (DQS) uchun qatlanishsizlik sharti olingan: $b/r \geq 3\cdot\tan(\theta/4)$. Hisoblash tajribasi uchala natijani tasdiqlaydi va an’anaviy global hajm mezoni bo’g’im nuqsonini 5–22 barobar kam ko’rsatishini aniqlaydi.

Аннотация. Результат автоматического риггинга стилизованных персонажей обычно оценивается визуально, что делает его невоспроизводимым. Предложена система геометрических критериев качества скиннинга, чувствительных к дефектам сустава: локальное отклонение площади сечения, непрерывность кривизны по двугранным углам (складки и перегибы), а также хаусдорфова и среднеквадратичная погрешности относительно эталона. Для двухкостного сустава выведены замкнутые выражения потери сечения при линейном смешивании (LBS): $\cos(\theta/2)$ при сгибе и $\cos^2(\theta/2)$ при скручивании. Для скиннинга на дуальных кватернионах (DQS) получено условие отсутствия перегибов $b/r \geq 3\cdot\tan(\theta/4)$. Вычислительный эксперимент подтверждает все три результата и показывает, что глобальный объём занижает дефект сустава в 5–22 раза.

Keywords: skeletal animation, linear blend skinning, dual quaternion skinning, deformation quality metric, volume preservation, curvature continuity, stylized characters, automatic rigging.

1. Introduction

Skeleton-driven skinning remains the standard way to deform characters in film, games and interactive media [12, 17]. Linear blend skinning was introduced by Magnenat-Thalmann, Laperrière and Thalmann in 1988 [1]. Its best-known artefacts, the collapsing elbow and the “candy-wrapper” twist, were analysed by Lewis, Cordner and Fong [2] and by Mohr and Gleicher [6]. Dual quaternion skinning by Kavan et al. [3] removed the collapse by blending rigid transformations. Later methods added elastic correction [4] and

optimized centres of rotation [5]; a systematic overview is given in the SIGGRAPH course of Jacobson et al. [7].

Automatic rigging research has moved quickly as well. Template-based and learning-based systems place a skeleton and compute weights without an artist. The quality of such systems, however, is still reported mostly through images and user studies. When a numerical measure is given, it is usually global: total volume change or mean vertex error over the whole mesh. A joint occupies a small part of the surface, so a global average dilutes exactly the defect that matters. For stylized characters with short, thick limbs this problem is sharper, because the joint zone is large relative to the limb and the artefacts are strong.

The local school of engineering geometry at TUIT provides the mathematical base used here: spatial transformations, polygonal meshes and spline surfaces [13]. The applied direction of 3D character animation at Uzbek universities has concentrated on facial modelling and blend shapes [14] and on practical rigging workflows [15, 16]. A reproducible geometric measure of body deformation quality has not been proposed in this line of work.

The aim of this paper is to define criteria that are local to the joint, invariant to limb length, reproducible by a third party, and to use them to derive design rules for automatic skinning. The contributions are:

1. A criterion set consisting of the local cross-section deviation, the crease and fold counts from dihedral angles, and reference-based Hausdorff and RMS errors.

2. Closed-form cross-section loss of LBS at a two-bone joint for bending and twisting.

3. A closed-form fold-free condition for DQS linking the blending-zone width to the limb radius and joint angle.

4. A computational experiment that verifies the analysis and quantifies the bias of the global volume metric.

2. Problem statement

Let a character surface be a triangle mesh M = (V, F) with rest positions v_i. Each vertex has skinning weights w_ij ≥ 0 over bones j, with Σ_j w_ij = 1, and each bone has a rigid transformation T_j = (R_j, t_j). LBS computes the deformed position as

$$v_i' = \sum_j w_{ij} \left(R_j v_i + t_j\right)$$

DQS converts each T_j to a unit dual quaternion q̂_j, blends them and normalizes [3]:

$$v_i' = \hat{q}_i \, v_i \, \hat{q}_i^*, \qquad \hat{q}_i = \frac{\sum_j w_{ij} \, \hat{q}_j}{\| \sum_j w_{ij} \, \hat{q}_j \|}$$

We study the model joint used throughout the skinning literature: a straight limb of radius r along the x-axis, parent bone fixed, child bone rotated by angle θ about the joint at x = 0. The child weight changes from 0 to 1 over a blending zone [−b, b] according to the smoothstep profile w(x) = 3t² − 2t³, t = (x + b)/(2b). The ratio r/L of radius to segment length describes the stylization of the limb: about 0.1 for realistic proportions and 0.3–0.45 for cartoon characters.

3. Proposed criteria

Criterion 1: local cross-section deviation. For every cross-section ring s in the joint zone we compute its polygon area A(s) in the rest pose and A’(s) after deformation:

$$\delta_A = \max_{s \in Z} \left| \frac{A'(s)}{A(s)} - 1 \right|$$

The maximum over the zone Z = [−2b, 2b] makes the value independent of limb length. For comparison we also report the global volume deviation δ_V = (V’ − V)/V computed from signed tetrahedra.

Criterion 2: curvature continuity. For every interior edge e we compute the dihedral angle α_e between the normals of its two triangles. Let α_0 be the largest dihedral angle of the rest mesh. We count

$$N_c = \#\{e: 3\alpha_0 < \alpha_e \le 90°\}, \qquad N_f = \#\{e: \alpha_e > 90°\}$$

N_c counts creases, visible as sharp ridges; N_f counts folds, where the surface turns over itself. Both are zero for a correct deformation. Unlike discrete curvature estimators [9, 10], dihedral angles need no smoothing and give the same result in any implementation.

Criterion 3: reference error. When an etalon deformation exists, for example a rig made by an artist, we use the RMS vertex error and the Hausdorff distance [11], both divided by the limb radius r so that characters of different size are comparable.

4. Analytical results

Proposition 1 (LBS cross-section loss). At the centre of the blending zone, w = 1/2. Under LBS the component of a cross-section point perpendicular to the rotation axis is multiplied by |(1 − w) + w·e^{iθ}|, which at w = 1/2 equals cos(θ/2). For bending, only one direction of the section lies in the rotation plane, so

$$\frac{A'}{A}\Big|_{\text{bend}} = \cos\frac{\theta}{2}, \qquad \frac{A'}{A}\Big|_{\text{twist}} = \cos^2\frac{\theta}{2}$$

For twisting both directions lie in the rotation plane, which gives the square. At θ = 180° the section area becomes zero: this is the candy-wrapper effect. The loss does not depend on r, b or L.

Proposition 2 (fold-free condition for DQS). DQS maps each section rigidly, so the only defect left is the inner side of the bend folding onto itself. The rotation angle along the axis is φ(x) = Φ(w(x)), where Φ is the angle of the normalized blend of the identity and the child rotation quaternions [3, 8]:

$$\Phi(w) = 2\arctan\frac{w\sin(\theta/2)}{(1-w) + w\cos(\theta/2)}, \qquad \Phi'(1/2) = 4\tan\frac{\theta}{4}$$

The fibre on the inner side at distance r from the axis is stretched by the factor 1 − r·φ’(x). It folds when this factor reaches zero. The smoothstep profile has maximal slope w’ = 3/(4b) at the centre, so the surface stays fold-free when

$$\frac{b}{r} \geq 3\tan\frac{\theta}{4}$$

For θ = 90° this gives b ≥ 1.24·r; for θ = 150°, b ≥ 2.30·r. The condition is an adaptive rule: the blending zone must scale with the local limb radius, not with the bone length. A fixed zone that works for a thin realistic arm will fold on a thick stylized one.

5. Computational experiment

Setup. Closed tube meshes of length 2 (two segments of length L = 1) were built with 200 rings of 64 vertices and capped ends: 12,802 vertices and 25,600 triangles. Radius ratios r/L were 0.10, 0.20, 0.30 and 0.45. Bending angles were 30°–150°, twisting angles 45°–180°. LBS and DQS were implemented in Python (NumPy, SciPy) directly from the formulas in Section 2. Timing was measured on one Intel Xeon 2.8 GHz core. The full code and result files are available from the author.

Result 1: global volume hides the joint defect. Table 1 shows LBS at θ = 90° bending with b = r. The local loss is 29.2–29.3% for every radius, as Proposition 1 predicts (1 − cos 45° = 29.3%). The global volume loss is only 1.3–5.8% and changes with the radius, although the defect itself does not.

r/L	Global volume δ_V, %	Local section loss δ_A, %	Theory 1 − cos(θ/2), %	Ratio δ_A / δ_V
0.10	−1.30	−29.1	−29.3	22.4
0.20	−2.58	−29.2	−29.3	11.3
0.30	−3.86	−29.3	−29.3	7.6
0.45	−5.79	−29.3	−29.3	5.1

Table 1. LBS, bending 90°, blending half-width b = r.

For twisting the measured local loss at r/L = 0.30 was 14.6%, 50.0%, 85.3% and 99.9% for 45°, 90°, 135° and 180°. The theoretical values 1 − cos²(θ/2) are 14.6%, 50.0%, 85.4% and 100%.

Result 2: volume criteria do not detect DQS folding. With b = r and θ = 90°, DQS keeps the global volume within 0.02% and the local section within 0.1% for all radii. Yet the dihedral criterion finds 90–210 creases and 6–14 folded edges on the inner side of the joint. The same holds at 120° and 150°. A metric based on volume alone would rate this deformation as perfect.

Result 3: the fold-free condition. For each angle the smallest b/r with no folds was found by a search with step 0.05, for r/L = 0.10, 0.30 and 0.45.

Joint angle θ	Predicted b/r = 3·tan(θ/4)	Measured minimum b/r, DQS (all three radii)	Measured minimum b/r, LBS (r/L = 0.30)
60°	0.80	0.85	0.90
90°	1.24	1.25	1.50
120°	1.73	1.75	2.60
150°	2.30	2.35	not reached up to 5.0

Table 2. Fold-free blending width.

The measured DQS threshold is the first sampled value above the prediction in every case, and it is identical for all three radii, which confirms that the condition depends only on the ratio b/r. LBS needs wider zones and cannot avoid folding at 150°, because its section collapses (Proposition 1).

Result 4: cost. One LBS evaluation of the 12,802-vertex mesh took 0.5–0.7 ms and one DQS evaluation 4.0–5.3 ms in unoptimized NumPy code. All criteria are computed in a single pass over edges and rings and add less than the cost of one deformation.

6. Discussion

The experiment shows that the choice of metric changes the conclusion. Global volume reports a 3.9% loss for a joint that has lost 29% of its section, and reports no problem for a DQS joint that has folded. Local and curvature criteria together detect both defects, and they do it with numbers that another researcher can reproduce from the mesh alone.

The fold-free condition turns an evaluation result into a design rule. An adaptive skinning framework can compute the limb radius at each joint from the mesh and the expected range of motion from the rig, and set the blending width to at least $3 \cdot \tan(\theta_max/4) \cdot r$. This is the geometric basis for the adaptive weight algorithm in the next stage of the research.

The analysis uses a straight limb of constant circular section and a smoothstep profile. Real limbs taper and have non-circular sections; for them r should be read as the local inscribed radius on the inner side of the bend. For other weight profiles the constant 3/4 is replaced by the maximal slope of the profile.

7. Conclusion

A set of geometric criteria for skinning quality has been proposed and justified. The local cross-section deviation is independent of limb length and matches the analytic loss of LBS exactly: $\cos(\theta/2)$ for bending and $\cos^2(\theta/2)$ for twisting. The dihedral criterion detects folds that volume criteria miss. The derived condition $b/r \geq 3 \cdot \tan(\theta/4)$ predicts the fold-free blending width of DQS within one sampling step. Together these results give a reproducible way to compare automatic rigging methods for stylized characters.

References

1. Magnenat-Thalmann N., Laperrière R., Thalmann D. Joint-dependent local deformations for hand animation and object grasping // Proceedings of Graphics Interface ’88. 1988. P. 26–33.

2. Lewis J.P., Cordner M., Fong N. Pose space deformation: a unified approach to shape interpolation and skeleton-driven deformation // Proceedings of ACM SIGGRAPH 2000. P. 165–172.

3. Kavan L., Collins S., Žára J., O’Sullivan C. Geometric skinning with approximate dual quaternion blending // ACM Transactions on Graphics. 2008. Vol. 27, No. 4. Article 105.

4. Kavan L., Sorkine O. Elasticity-inspired deformers for character articulation // ACM Transactions on Graphics. 2012. Vol. 31, No. 6. Article 196.

5. Le B.H., Hodgins J.K. Real-time skeletal skinning with optimized centers of rotation // ACM Transactions on Graphics. 2016. Vol. 35, No. 4. Article 37.

6. Mohr A., Gleicher M. Building efficient, accurate character skins from examples // ACM Transactions on Graphics. 2003. Vol. 22, No. 3. P. 562–568.

7. Jacobson A., Deng Z., Kavan L., Lewis J.P. Skinning: real-time shape deformation // ACM SIGGRAPH 2014 Courses. 2014.

8. Shoemake K. Animating rotation with quaternion curves // Computer Graphics (SIGGRAPH ’85). 1985. Vol. 19, No. 3. P. 245–254.

9. Botsch M., Kobbelt L., Pauly M., Alliez P., Lévy B. Polygon Mesh Processing. Natick: A K Peters, 2010. 250 p.

10. Meyer M., Desbrun M., Schröder P., Barr A.H. Discrete differential-geometry operators for triangulated 2-manifolds // Visualization and Mathematics III. Berlin: Springer, 2003. P. 35–57.

11. Aspert N., Santa-Cruz D., Ebrahimi T. MESH: measuring errors between surfaces using the Hausdorff distance // Proceedings of IEEE ICME. 2002. P. 705–708.

12. Parent R. Computer Animation: Algorithms and Techniques. 3rd ed. Waltham: Morgan Kaufmann, 2012.

13. Nazirov Sh.A., Nuraliyev F.M., To’rayev B.Z. Kompyuter grafikasi va dizayn: o’quv qo’llanma. Toshkent: Fan va texnologiya, 2015. 256 b. ISBN 978-9943-990-80-7.

14. Kayumova G., Boymurodov B. Uch o’lchamli personajlarning yuz holatini modellashtirish va animatsiyalashning zamonaviy yondashuvlari // Al-Farg’oniy avlodlari. 2025. Vol. 1, No. 3. P. 151–158. DOI: 10.5281/zenodo.17295640.

15. O’tkirbekova M.R. Development and implementation of optimization and automation methods for creating and animating 3D characters using frameworks for controlling bone structures in Blender // Образование наука и инновационные идеи в мире. 2025. No. 78, Part 1. P. 373–378.

16. O’tkirbekova M.R. Algorithmic optimization of bone and animation structures in 3D animation: development of a universal framework for Blender // International Conference on Innovative Development of Education. 2025. No. 1. P. 34–36. DOI: 10.5281/zenodo.17268419.

17. Magnenat-Thalmann N., Thalmann D. (eds.) Handbook of Virtual Humans. Chichester: John Wiley & Sons, 2004.